Stark-Induced Electric Dipole Amplitudes for Hyperfine Transitions

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Stark-induced electric dipole amplitudes between states of the same nominal parity can be important in experiments to observe parity nonconservation in atoms. The Stark-induced E1 amplitudes are expressed in terms of an irreducible spherical-tensor decomposition. This formalism is applied to the specific case of transitions between hyperfine sublevels of a single atomic state. It is shown that in the ground states of alkali atoms, such transitions are suppressed by many orders of magnitude relative to naive expectations.

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I. INTRODUCTION

The first definitive observation of a nuclear spin-dependent (NSD) contribution to atomic parity nonconservation (PNC) was recently reported [1]. This effect arises primarily because of an electromagnetic interaction between atomic electrons and the nucleus. In particular, because of PNC interactions *within* the nucleus, the nucleus acquires a P-odd electromagnetic multipole moment known as the anapole moment [2,3]. Measurements of atomic NSD-PNC amplitudes can thus be interpreted in terms of the nuclear anapole moment. The nuclear anapole moment itself can be related to the strengths of various hadronic PNC couplings. These couplings are in general poorly determined, and the available data are in some cases contradictory. Thus, there is considerable interest in additional measurements of atomic NSD-PNC, as a means to probe hadronic neutral-weak interactions [4,5].

NSD-PNC effects are extremely small, and their observation requires both extreme statistical sensitivity and careful control over minute systematic effects. Gorshkov *et al.* have proposed a novel technique for observing NSD-PNC [6]. They suggest using a strong microwave electric field to drive PNC-induced electric dipole (E1) transitions between hyperfine sublevels of a single atomic state. Interference between this E1 transition amplitude and a (parity-allowed) M1 amplitude driven by a weak microwave magnetic field (coherent with the electric field) gives rise to a PNC observable. This technique offers the promise for unprecedented statistical sensitivity. However, Gorshkov *et al.* give only a limited and qualitative analysis of possible systematic effects in the measurement they propose.

Further consideration of the proposal of Ref. [6] has led us to investigate the phenomenon of Stark-induced (SI) E1 transitions between hyperfine sublevels. These arise in the presence of a DC electric field, which mixes states of opposite parity into the initial and final states; thus E1 transitions can be induced between one nominal state and the admixed component of the other state [7]. Such transitions can be of importance in PNC measurements for two reasons. First, the presence of stray electric fields leads to SI-E1 transition amplitudes between hyperfine sublevels, which can mimic the effect of the PNC-induced E1 amplitudes; the size of the SI amplitudes relative to the PNC-induced amplitudes determines the extent to which stray fields must be controlled. Second, by deliberately applying a large electric field, the SI-E1 amplitude could in principle be used as the primary parity-allowed amplitude, against which the PNC amplitude can interfere [7]. This technique gives excellent control over a variety of systematic effects, and indeed is the principle of the measurement of Ref. [1], in which the optical 6s-7s transition in Cs was studied.

We show here that in the particular case of alkali atoms (as discussed in Ref. [6]), the SI-E1 amplitudes between hyperfine sublevels of the ground state are suppressed by many orders of magnitude relative to naive expectations. This conclusion means that it is impossible in practice to use the SI amplitudes as the primary parity-allowed amplitude in experiments of the type proposed by Gorshkov *et al.* On the other hand, it also means that systematic effects due to stray electric fields in such experiments are negligibly small under reasonable conditions.

II. GENERAL ARGUMENT FOR THE SUPPRESSION OF SI-E1 HYPERFINE TRANSITIONS

The SI-E1 transition $a \to b$ in a constant field $\mathbf{E}$ and time-varying field $\boldsymbol{\varepsilon}$ has amplitude

$$W_{ba} = \sum_{q,q'} (-1)^{q+q'} A_{q,q'} E_{-q} \varepsilon_{-q'}, \tag{1}$$

where

$$A_{q,q'} = e^2 \sum_n \left(\frac{\langle b|r_q|n\rangle\langle n|r_{q'}|a\rangle}{E_b - E_n} + \frac{\langle b|r_{q'}|n\rangle\langle n|r_q|a\rangle}{E_a - E_n} \right), \tag{2}$$

and the subscripts q, q' refer to spherical vector components.

The SI-E1 transition can be usefully understood as a special case of a two-photon transition, with one photon at zero frequency. For electric dipole interactions, each photon has orbital angular momentum $L = 0$ with respect to the atom. Thus, since each photon has spin $S = 1$, the two-photon system must have total angular momentum $J_{\rm tot} = S_{\rm tot} = 0, 1$, or 2. These three cases correspond to scalar ($\alpha_{\rm St}$), vector ($\beta_{\rm St}$), and tensor ($\gamma_{\rm St}$) transition amplitudes, respectively. We are primarily interested here in transitions between sublevels of the $s_{1/2}$ ground state of an alkali atom. For this case, each of the three possible amplitudes is strongly suppressed (or vanishes) relative to the naive estimate of the amplitude: $A \sim e^2 a_0^2/\mathcal{R} \sim a_0^3$ (where a_0 is the Bohr radius and $\mathcal{R}$ is the Rydberg constant). These suppressions can be understood in terms of selection rules for two-photon transitions, which were derived in Ref. [8]. We begin by considering these (approximate) selection rules for the specific case of hyperfine transitions, and estimate the residual values of the SI-E1 amplitudes.

First, we note that the scalar amplitude $\alpha_{\rm St}$ vanishes for hyperfine transitions. This is a trivial consequence of the fact that the scalar amplitude cannot change any angular quantum numbers, and thus cannot contribute to any transitions between sublevels of the same state. This argument fails when the principal quantum number changes; thus, for the well-studied 6s-7s transition in Cs, $\alpha_{\rm St}(6s - 7s) = 269\ a_0^3$ [9]. (This is somewhat larger than the naive estimate given above because the radial matrix elements in Cs and other alkalis are $\sim 5a_0$, and the energy denominators are $\sim 0.1\mathcal{R}$.)

Next, we show that for hyperfine transitions, the vector amplitude $\beta_{\rm St}$ is suppressed by a factor $\sim \frac{m_e}{m_p} Z\alpha^2$, where α is the fine-structure constant. This suppression can be understood as a consequence of Bose statistics for photons. For $S_{\rm tot}$=1, the spin component of a two-photon wavefunction is antisymmetric under interchange of the two photons. Thus, the space-time component of the wavefunction must also be antisymmetric under interchange, in order to satisfy Bose statistics. In the electric-dipole approximation, where the spatial part of the wavefunction is constant, the total wavefunction must therefore vanish if the photons have the same energy. (This statement is closely related to the Landau-Yang theorem of high-energy physics, which states that the decay of a J=1 particle into two photons is forbidden [10,11]; this result has also been derived for atomic two-photon transitions [8].) In the limit where the frequencies of the two photons are close but not identical, the vector part of the two-photon transition amplitude does not vanish, but is suppressed by a factor $\sim \frac{\hbar\omega_1 - \hbar\omega_2}{E_v - E_r}$, where $\hbar\omega_i$ is the energy of the ith photon, and $E_v - E_r$ is the difference in energy between a virtual intermediate state and a real intermediate state [12]. For the case of SI-E1 transitions between hyperfine sublevels of the same state, $\hbar\omega_1 - \hbar\omega_2 = \hbar\omega = E_{\rm hfs}$ (the hyperfine splitting energy), $E_v = E_{\rm hfs}$ (or 0), and $E_r = E_{\rm el}$ (the energy splitting between electronic states). Thus, for the case of interest here, $\beta_{\rm St}$ is suppressed by an additional factor $\sim \frac{E_{\rm hfs}}{E_{\rm el}} \sim \frac{m_e}{m_p} Z\alpha^2$ (see e.g. [13]). Clearly, this suppression is absent for the vector component of optical SI-E1 transitions such as the 6s-7s transition in Cs.

For the particular case of transitions between hyperfine sublevels of the ground state of an alkali atom, the vector amplitude is suppressed by an additional factor of $\sim Z^2\alpha^2$. In the absence of both electron and nuclear spin, $\beta_{\rm St}$ must vanish for any $s - s'$ $(0 - 0')$ transition. However, the addition of electron spin alone is sufficient to allow this amplitude, since then the transition is of the type $\frac{1}{2} - \frac{1}{2}'$. (Nuclear spin alone will also induce a nonzero value of $\beta_{\rm St}$, but this is a much smaller effect.) Since for any $s_{1/2} - s'_{1/2}$ transition $\beta_{\rm St}$ explicitly relies on the presence of electron spin, it must be that $\beta_{\rm St}$ is suppressed relative to the naive estimate by $\sim \frac{E_{\rm fs}}{E_v - E_r} \sim \frac{E_{\rm fs}}{E_{\rm el}} \sim Z^2\alpha^2$, where $E_{\rm fs}$ is a fine-structure energy splitting (see e.g. [13]). This suppression is indeed present for the 6s-7s transition in Cs [7], where the ratio of vector to scalar amplitudes is $\beta_{\rm St}/\alpha_{\rm St} = 0.1$ [14]. For the hyperfine transitions of interest here, the vector amplitude is thus suppressed relative to its naive value by an overall factor $\sim \frac{m_e}{m_p} Z^3\alpha^4$.

Finally, we consider the tensor part $\gamma_{\rm St}$ of the SI-E1 amplitude. For $s - s'$ transitions this amplitude vanishes even in the presence of electron spin (i.e., for a $\frac{1}{2} - \frac{1}{2}'$ transition); its presence relies explicitly on the presence of nuclear spin. Thus, $\gamma_{\rm St}$ must be suppressed relative to the naive value by a factor $\sim \frac{E_{\rm hfs}}{E_v - E_r} \sim \frac{E_{\rm hfs}}{E_{\rm el}} \sim \frac{m_e}{m_p} Z\alpha^2$. Let us make this statement more explicit by noting that, for any nuclear spin $I \geq 1$, nuclear spin alone is sufficient to induce a non-zero tensor amplitude. (For all stable alkali-atom nuclei, $I \geq 1$.) Since electron spin is neither necessary nor sufficient to produce a tensor amplitude, we expect that its presence should not significantly alter the magnitude of $\gamma_{\rm St}$. Rather, the tensor amplitude can be understood from the hypothetical case of an atom with nuclear spin, but no electron spin. In this case, the only relevant hyperfine splitting is that due to the interaction of p-states with the

nuclear magnetic dipole and electric quadrupole moments. Thus, $\gamma_{\rm St}$ is suppressed relative to its naive value by a factor of $\sim \frac{E_{\rm hfs}(p)}{E_{\rm el}}$. This argument holds even for optical $s-s'$ transitions such as the 6s-7s transition in Cs; for this reason, the amplitude $\gamma_{\rm St}$ is negligibly small for this transition, and is always ignored. We note in passing that the suppression of $\gamma_{\rm St}$ is not present at all for transitions between hyperfine sublevels of any electronic state with $J \geq 1$. However, we confine ourselves to the particular case of hyperfine transitions in alkali-atom ground states.

III. EXPRESSIONS FOR SCALAR, VECTOR AND TENSOR POLARIZABILITIES

We now discuss specific calculations of the SI-E1 amplitudes. For our purposes it is convenient to rewrite the general second-rank tensors $A_{q,q'}$ and $E_{-q}\varepsilon_{-q'}$ of (1) and (2) in terms of their irreducible tensor components A^K_Q and $(E\otimes\varepsilon)^K_{-Q}$. We use the standard transformations [13], e.g.

$$A^K_Q = (-1)^Q\sqrt{2K+1}\sum_{q,q'}\begin{pmatrix} 1 & 1 & K \\ -q & -q' & Q\end{pmatrix} A_{q,q'}, \tag{3}$$

and

$$E_{-q}\varepsilon_{-q'} = \sum_{K,Q}(-1)^Q\sqrt{2K+1}\begin{pmatrix} 1 & 1 & K \\ -q & -q' & Q\end{pmatrix}(E\otimes\varepsilon)^K_{-Q} \tag{4}$$

so that the transition probability (1) is written as the contraction of irreducible tensor components (as was done for the general case of two-photon transitions in Ref. [8]):

$$W_{ba} = \sum_{K,Q}(-1)^Q A^K_Q (E\otimes\varepsilon)^K_{-Q}. \tag{5}$$

We further rewrite the irreducible tensor component A^K_Q in terms of A^K, the reduced amplitude of rank K :

$$A^K_Q = (-1)^{J_b-M_b}\begin{pmatrix} J_b & K & J_a \\ -M_b & Q & M_a\end{pmatrix} A^K. \tag{6}$$

In the general case the transition amplitude is fully described by three independent reduced amplitudes with $K = 0,1,2$.

The SI-E1 transition amplitude has traditionally been written in terms of the quantities $\alpha_{\rm St}$, $\beta_{\rm St}$, and $\gamma_{\rm St}$, such that

$$W_{ba} = \alpha_{\rm St}\mathbf{E}\cdot\boldsymbol{\varepsilon} + \boldsymbol{\beta}_{\rm St}\cdot\mathbf{E}\times\boldsymbol{\varepsilon} + \gamma^{i,k}_{\rm St}(\frac{1}{2}E_i\varepsilon_k + \frac{1}{2}E_k\varepsilon_i - \frac{1}{3}\mathbf{E}\cdot\boldsymbol{\varepsilon}\delta_{i,k}). \tag{7}$$

The quantities $\alpha_{\rm St}$, $\beta_{\rm St}$, and $\gamma_{\rm St}$ are simply related to the reduced tensor amplitudes in Eq. (6); for instance,

$$\alpha_{\rm St} = \frac{-1}{\sqrt{3(2J_a+1)}}A^0, \quad \beta^q_{\rm St} = \frac{i}{\sqrt{2}}(-1)^{J_b-M_b}\begin{pmatrix} J_b & 1 & J_a \\ -M_b & q & M_a\end{pmatrix} A^1. \tag{8}$$

From here on we will use the reduced amplitudes A^K rather than $\alpha_{\rm St}$, $\beta_{\rm St}$, and $\gamma_{\rm St}$, since the irreducible tensor formalism makes it possible to write expressions in a general form for all K.

At this stage let us assume that there is no nuclear spin and levels are described by quantum numbers J_i, M_i. Then we can replace the matrix elements of r_q in (2) in terms of reduced matrix elements, e.g.

$$\langle n|r_q|a\rangle = (-1)^{J_n-M_n}\langle n||r||a\rangle\begin{pmatrix} J_n & 1 & J_a \\ -M_n & q & M_a\end{pmatrix}. \tag{9}$$

Using Eqs. (2), (3), (6), and (9), we obtain an expression for A^K, with the sum (over q, q', and M_n) of the product of the three 3j-symbols reduced to a 6j-symbol:

$$A^K = \sqrt{2K+1}\sum_n(-1)^{J_a+J_n}\begin{Bmatrix} J_n & J_b & 1 \\ K & 1 & J_a\end{Bmatrix}\langle n||r||b\rangle\langle n||r||a\rangle\left(\frac{1}{E_b-E_n}+\frac{(-1)^K}{E_a-E_n}\right). \tag{10}$$

A. SI-E1 amplitude for nonzero nuclear spin

We now include quantum numbers F, M and I. First we can simply substitute $J, M \to F, M$ in (10):

$$A^K = \sqrt{2K+1}\sum_n (-1)^{F_a+F_n} \left\{ \begin{array}{ccc} F_n & F_b & 1 \\ K & 1 & F_a \end{array} \right\} \times \\ \langle n, F_n||r||b, F_b\rangle\langle n, F_n||r||a, F_a\rangle \left(\frac{1}{E_b - E_n} + \frac{(-1)^K}{E_a - E_n}\right). \tag{11}$$

The dependence of the reduced matrix element on the quantum numbers F is described by the expression:

$$\langle n, F_n||r||a, F_a\rangle = (-1)^{J_n+I+F_a+1}\sqrt{(2F_n+1)(2F_a+1)} \left\{ \begin{array}{ccc} J_n & F_n & I \\ F_a & J_a & 1 \end{array} \right\} \langle n||r||a\rangle, \tag{12}$$

so that

$$A^K = (-1)^{2I+2F_a+F_b}\sqrt{(2K+1)\,(2F_n+1)(2F_a+1)}\sum_n (-1)^{2J_n+F_n}(2F_n+1) \times \\ \left\{ \begin{array}{ccc} F_n & F_b & 1 \\ K & 1 & F_a \end{array} \right\} \left\{ \begin{array}{ccc} J_n & F_n & I \\ F_b & J_b & 1 \end{array} \right\} \left\{ \begin{array}{ccc} J_n & F_n & I \\ F_a & J_a & 1 \end{array} \right\} \langle n||r||b\rangle\langle n||r||a\rangle \left(\frac{1}{E_b - E_n} + \frac{(-1)^K}{E_a - E_n}\right).$$

Up to this point, our expressions have been perfectly general. From now on, we introduce expressions that are specific to single valence-electron atoms. In particular, we write the reduced matrix elements of r explicitly in terms of the single-electron angular momenta l (orbital) and j (total):

$$\langle n||r||a\rangle = (-1)^{j_a+l_{an}-1/2}\sqrt{(2j_n+1)(2j_a+1)l_{an}} \left\{ \begin{array}{ccc} l_n & j_n & \frac{1}{2} \\ j_a & l_a & 1 \end{array} \right\} R_{n,a}, \tag{13}$$

where $l_{an} \equiv \max(l_a, l_n)$ and $R_{n,a}$ is the radial integral. Combining these two equations gives

$$A^K = -\sqrt{(2K+1)(2F_a+1)(2F_b+1)(2j_a+1)(2j_b+1)}\sum_n C^K_{F_n,j_n,l_n} D^K_n, \tag{14}$$

$$C^K_{F_n,j_n,l_n} \equiv (-1)^{F_n+F_b+j_a+j_b}(2F_n+1)(2j_n+1) \times \\ \left\{ \begin{array}{ccc} F_n & F_b & 1 \\ K & 1 & F_a \end{array} \right\} \left\{ \begin{array}{ccc} j_n & F_n & I \\ F_a & j_a & 1 \end{array} \right\} \left\{ \begin{array}{ccc} j_n & F_n & I \\ F_b & j_b & 1 \end{array} \right\} \left\{ \begin{array}{ccc} l_n & j_n & \frac{1}{2} \\ j_a & l_a & 1 \end{array} \right\} \left\{ \begin{array}{ccc} l_n & j_n & \frac{1}{2} \\ j_b & l_b & 1 \end{array} \right\}, \tag{15}$$

$$D^K_n \equiv (-1)^{l_{an}+l_{bn}}\sqrt{l_{an}l_{bn}}R_{n,a}R_{n,b}\left(\frac{1}{E_b - E_n} + \frac{(-1)^K}{E_a - E_n}\right). \tag{16}$$

Note that $D^1_n \propto E_a - E_b$; this is the suppression discussed earlier, due to Bose statistics for photons.

B. Summation over F_n and j_n

The general arguments for the suppression of the hyperfine SI-E1 amplitudes suggest that there are cancellations in the sums over quantum numbers F_n and j_n. To see this explicitly, note that in the nonrelativistic limit, D^K_n does not depend on j_n or F_n. Application of the Racah-Eliot relations [15] to the sums over F_n and j_n gives:

$$\sum_{F_n} C^K_{F_n,j_n,l_n} = (-1)^{F_a+I+K+j_n}(2j_n+1) \times \\ \left\{ \begin{array}{ccc} K & j_a & j_b \\ I & F_b & F_a \end{array} \right\} \left\{ \begin{array}{ccc} K & j_a & j_b \\ j_n & 1 & 1 \end{array} \right\} \left\{ \begin{array}{ccc} l_n & j_n & \frac{1}{2} \\ j_a & l_a & 1 \end{array} \right\} \left\{ \begin{array}{ccc} l_n & j_n & \frac{1}{2} \\ j_b & l_b & 1 \end{array} \right\}, \tag{17}$$

$$\sum_{j_n,F_n} C^K_{F_n,j_n,l_n} = (-1)^{F_a+I+j_a+j_b+l_n-1/2} \left\{ \begin{array}{ccc} K & j_a & j_b \\ I & F_b & F_a \end{array} \right\} \left\{ \begin{array}{ccc} K & l_a & l_b \\ \frac{1}{2} & j_b & j_a \end{array} \right\} \left\{ \begin{array}{ccc} K & l_a & l_b \\ l_n & 1 & 1 \end{array} \right\}. \tag{18}$$

Note that both expressions turn to zero for $K > j_a + j_b$; this is equivalent to the vanishing of a $\frac{1}{2}-\frac{1}{2}'$ tensor amplitude in the absence of nuclear spin. In addition expression (18) turns to zero for $K > l_a + l_b$. This is equivalent to the vanishing of $s - s'$ vector (tensor) amplitudes in the absence of electron (nuclear) spin.

Equations (17) and (18) make it possible to simplify the general expression (14). If we neglect the dependence of D_n^K on F_n , we can use the sum (17). If we also neglect the dependence of D_n^K on j_n, we can use the sum (18). This yields the following expressions:

$$A^K = -\sqrt{(2K+1)(2F_a+1)(2F_b+1)(2j_a+1)(2j_b+1)} \left\{ \begin{array}{ccc} K & j_a & j_b \\ I & F_b & F_a \end{array} \right\} \times$$
$$\sum_n (-1)^{F_a+I+K+j_n} (2j_n+1) \left\{ \begin{array}{ccc} K & j_a & j_b \\ j_n & 1 & 1 \end{array} \right\} \left\{ \begin{array}{ccc} l_n & j_n & \frac{1}{2} \\ j_a & l_a & 1 \end{array} \right\} \left\{ \begin{array}{ccc} l_n & j_n & \frac{1}{2} \\ j_b & l_b & 1 \end{array} \right\} D_n^K, \tag{19}$$

$$A^K = -\sqrt{(2K+1)(2F_a+1)(2F_b+1)(2j_a+1)(2j_b+1)} \left\{ \begin{array}{ccc} K & j_a & j_b \\ I & F_b & F_a \end{array} \right\} \left\{ \begin{array}{ccc} K & l_a & l_b \\ \frac{1}{2} & j_b & j_a \end{array} \right\} \times$$
$$\sum_n (-1)^{F_a+I+j_a+j_b+l_n-1/2} \left\{ \begin{array}{ccc} K & l_a & l_b \\ l_n & 1 & 1 \end{array} \right\} D_n^K, \tag{20}$$

where D_n^K is given by (16). The first of these equations is correct up to the hyperfine corrections, while the second is correct only up to the fine structure corrections.

IV. SI-E1 AMPLITUDES FOR HYPERFINE TRANSITIONS IN ALKALI ATOMS

We now apply these formulae to the $F_a = I - \frac{1}{2} \to F_b = I + \frac{1}{2}$ transitions of the ground states $n_0 s_{1/2}$ of alkali atoms. As we have seen above, for such transition both vector and tensor amplitudes are strongly suppressed. Here we want to obtain accurate estimates of these amplitudes.

It is known that for all alkalis

$$|R_{n_0 p, n_0 s}| \gg |R_{np, n_0 s}|, \quad n > n_0. \tag{21}$$

Thus, the dominant contribution to the sums (14), (19) and (20) comes from the p shell with the principle quantum number n_0.

A. Vector amplitude

For the case of interest where $l_a = l_b = 0$, the nonrelativistic expression (20) gives zero value for the amplitude A^1; to get a correct value, we must use equation (19) which includes fine-structure corrections. In order to avoid cancellation in the sum over j_n we have to take into account the dependence of the factor D_n^1 on j_n, which can be written as:

$$D_n^1 = \bar{D}_n^1 + (j_n - l_n) d_n^1, \tag{22}$$

where $\bar{D}_n^1$ and d_n^1 do not depend on j_n. We have seen that the contributions of $j_n = \frac{1}{2}$ and $j_n = \frac{3}{2}$ cancel each other for $\bar{D}_n^1$. Then obviously for d_n^1 they should double each other, and we find:

$$A^1 = \frac{4}{9}(-1)^{2I} \sqrt{3I(I+1)} \left\{ \begin{array}{ccc} I - \frac{1}{2} & I + \frac{1}{2} & 1 \\ \frac{1}{2} & \frac{1}{2} & I \end{array} \right\} \sum_{np} d_{np}^1. \tag{23}$$

Note that the sum runs over states np and does not include summation over j_n.

Now we have to find d_{np}^1. It follows from (22) that $d_{np}^1 = D_{np_{3/2}}^1 - D_{np_{1/2}}^1$ and it is not zero because of the spin-orbit interaction $H_{\rm so}$. There are diagonal and off-diagonal contributions of $H_{\rm so}$ to d_{np}^1:

- The fine-structure splitting changes the energy denominators in (16). If we define $\Delta_{{\rm so},np} = E_{np_{3/2}} - E_{np_{1/2}}$ and note that for the hyperfine transition $E_b - E_a = A_{n_0 s}(I + \frac{1}{2})$, where $A_{n_0 s}$ is the hyperfine constant of the ground state, we get for d_n^1:

$$d_n^1 = -(2I+1) R_{np, n_0 s}^2 \frac{A_{n_0 s} \Delta_{{\rm so},np}}{(E_{n_0 s} - E_{np})^3}. \tag{24}$$

- The spin-orbit interaction changes the radial integrals in (16):

$$\delta R_{np_j,n_0s} = \sum_{n'\neq n} \frac{\langle np|H_{\rm so}|n'p\rangle R_{n'p,n_0s}}{E_{np}-E_{np'}}. \tag{25}$$

Parametrically both mechanisms give the same smallness for d_n^1, but numerically for alkali atoms the off-diagonal correction is suppressed. Indeed, for the dominant term $n = n_0$, the correction to the radial integral is numerically small because of the relation (21).

Thus the vector amplitude can be written:

$$A^1 = \frac{4}{9}(-1)^{2I+1}(2I+1)\sqrt{3I(I+1)} \left\{ \begin{array}{ccc} I-\frac{1}{2} & I+\frac{1}{2} & 1 \\ \frac{1}{2} & \frac{1}{2} & I \end{array} \right\} R^2_{n_0p,n_0s} \frac{A_{n_0s}\Delta_{{\rm so},np}}{(E_{n_0s}-E_{n_0p})^3}. \tag{26}$$

B. Tensor amplitude

Here both expressions (19) and (20) turn to zero and we have to return to the expressions (14)–(16). Because of the relation (21) we again can restrict the summation to the shell n_0p. Then the sum runs over $F = I \pm \frac{1}{2}$ and $j = \frac{1}{2}, \frac{3}{2}$. According to (17) the first sum vanishes because $j_a = j_b = \frac{1}{2}$, while (18) shows that the second sum vanishes because $l_a = l_b = 0$. That means that all four coefficients $C^2_{F_n,j_n,0}$ differ only by their signs. So, it is sufficient to calculate, for example, $C^2_{I-\frac{1}{2},\frac{1}{2},0}$:

$$C^2_{I-\frac{1}{2},\frac{1}{2},0} = \frac{2}{3}(-1)^{2I+1} I \left\{ \begin{array}{ccc} I-\frac{1}{2} & I+\frac{1}{2} & 1 \\ 2 & 1 & I-\frac{1}{2} \end{array} \right\} \left\{ \begin{array}{ccc} \frac{1}{2} & I-\frac{1}{2} & I \\ I-\frac{1}{2} & \frac{1}{2} & 1 \end{array} \right\} \left\{ \begin{array}{ccc} \frac{1}{2} & I-\frac{1}{2} & I \\ I+\frac{1}{2} & \frac{1}{2} & 1 \end{array} \right\}. \tag{27}$$

It is clear that in order to avoid cancellations we have to include hyperfine corrections to D_n^2 . Again, as in the case of vector amplitude, we can neglect the correction to the radial integrals and consider only corrections to the energy denominators. For the $n_0p_{3/2}$ state the quadrupole hyperfine structure should be included. That gives the following result:

$$A^2 = \frac{8}{3}(-1)^{2I+1} I(2I+1)\sqrt{5I(I+1)} \left\{ \begin{array}{ccc} I-\frac{1}{2} & I+\frac{1}{2} & 1 \\ 2 & 1 & I-\frac{1}{2} \end{array} \right\} \left\{ \begin{array}{ccc} \frac{1}{2} & I-\frac{1}{2} & I \\ I-\frac{1}{2} & \frac{1}{2} & 1 \end{array} \right\} \times$$
$$\left\{ \begin{array}{ccc} \frac{1}{2} & I-\frac{1}{2} & I \\ I+\frac{1}{2} & \frac{1}{2} & 1 \end{array} \right\} R^2_{n_0p,n_0s} \left[\frac{A_{n_0p_{1/2}}}{(E_{n_0s}-E_{n_0p_{1/2}})^2} + \frac{-A_{n_0p_{3/2}} + \frac{3B_{n_0p_{3/2}}}{(I(2I-1))}}{(E_{n_0s}-E_{n_0p_{3/2}})^2} \right], \tag{28}$$

where $A_{n_0p_j}$ and $B_{n_0p_{3/2}}$ are the hyperfine constants for the levels n_0p_j.

C. Numerical results

Equations (26) and (28) show that, in agreement with our general arguments, the vector amplitude $A^1 \sim \frac{\Delta_{{\rm hfs},n_0s}}{\Delta_{sp}} \frac{\Delta_{{\rm so},n_0p}}{\Delta_{sp}} a_0^3 \propto \frac{m_e}{m_p} Z^3\alpha^4$, while the tensor amplitude $A^2 \sim \frac{\Delta_{{\rm hfs},n_0p}}{\Delta_{sp}} a_0^3 \propto \frac{m_e}{m_p} Z\alpha^2$. This means that for light alkalis the tensor amplitude dominates; however, the vector amplitude grows more rapidly with Z and for heavy alkalis may even be larger than the tensor amplitude, since the hyperfine constant of the ground s state is much larger than the hyperfine constants of the p states.

To obtain numerical values of these amplitudes we need to know the radial integrals R_{n_0p,n_0s} and hyperfine constants A_{n_0s} and $A_{n_0p_j}$. They are given in Table I together with our results for the amplitudes A^1 and A^2. The uncertainty in these values arises primarily from the two approximations we have made, namely: neglecting intermediate p-states with $n > n_0$ in the sum of equation (14), and neglecting the j-dependence of radial matrix elements. We estimate that these approximations introduce errors of $\lesssim 10\%$.

As stated in the introduction, the motivation for this calculation was the effect of the SI-E1 amplitudes in experiments to measure PNC in hyperfine transitions. In order to make the SI-E1 amplitudes comparable to the parity-allowed M1 transitions between hyperfine sublevels (and thus to use the SI-E1 amplitudes as the primary parity-allowed amplitude), it is necessary that $A \sim \mu_B$. Even for Fr, where A attains its largest value, this is achieved

only for $E \sim 5 \times 10^9$ V/cm! Such a prospect is clearly unrealistic. On the other hand, it is also instructive to compare the magnitude of the SI-E1 amplitudes to the PNC-induced E1 amplitudes. Using the values for the PNC amplitudes for hyperfine transitions from Ref. [6] ($\kappa D_{Cs} \approx -3 \times 10^{-13} e a_0$, $\kappa D_K \approx -1 \times 10^{-14} e a_0$), we find that in both cases the largest Stark-induced amplitude is comparable to the PNC-induced amplitude only when the electric field $E \sim 3$ V/cm. Since stray fields well below this level are easily controlled, systematic effects due to uncontrolled SI-E1 amplitudes in experiments of the type proposed by Gorshkov *et al*. should not be a serious problem, even for the lighter alkalis.

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TABLE I. Input data and numerical values for SI-E1 amplitudes. Unless otherwise noted, data for Δ_{sp}, $\Delta_{\mathrm{so},n_0p}$, A_{n_0s}, $A_{n_0p_{1/2}}$, $B_{n_0p_{3/2}}$, and R_{n_0s,n_0p} are taken from Ref. [16].

		^{7}Li	^{23}Na	^{39}K	^{85}Rb	^{133}Cs	^{221} Fr
I		$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$	$\frac{5}{2}$ [a]
Δ_{sp}	(cm^{-1})	14904	16965	13014	12698	11456	13081 [b]
$\Delta_{\mathrm{so},n_0p}$	(cm^{-1})	0.34	17.2	57.9	238	554	1687 [b]
A_{n_0s}	(GHz)	0.402	0.886	0.231	1.01	2.30	6.21 [a]
$A_{n_0p_{1/2}}$	(MHz)	45.9	94.3	27.8	120.7	292	811 [b]
$A_{n_0p_{3/2}}$	(MHz)	-3.06	18.7	6.1	25.0	50.3	66.5 [b]
$B_{n_0p_{3/2}}$	(MHz)	-0.2	2.9	2.8	26.0	-0.4	-260 [b]
R_{n_0s,n_0p}	(a_0)	4.05	4.29	5.06	5.03	5.50	5.11 [c]
A^1	($10^{-6}a_0^3$)	0.0085	0.72	2.0	70	940	2900
A^2	($10^{-6}a_0^3$)	20	27	20	185	1050	1350

[a] Ref. [17]
[b] Ref. [18]
[c] Ref. [19]