Manifestation of nuclear spin-dependent P-odd electron–nucleon interaction in atomic ytterbium

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P-odd effects caused by the nuclear spin-dependent electron–nucleon interaction are considered. P-odd amplitudes are calculated for $^1S_0 \to {}^3D_{1,2}$ transitions in atomic ytterbium.

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1. Introduction

In the present paper we calculate the amplitude of nuclear spin-dependent electron–nucleus interaction for atomic ytterbium. Three effects contribute to this amplitude: the interaction of an electron with the nuclear anapole moment (AM), the electron–nucleon neutral current interaction, and the combined action of the nuclear spin-independent electron–nucleus weak interaction and the hyperfine interaction. The anapole moment was introduced by Zel'dovich [1] just after the discovery of parity violation. He pointed out that a particle should have a parity-violating electromagnetic form factor, in addition to the usual electric and magnetic form factors. The classical example is a toroidal coil with current. A toroidal electromagnetic current density **j** produces a magnetic field inside the torus. A magnetic field of the same configuration can occur in a nucleus if parity is violated. In the limit of a point-like nucleus the vector potential **A** corresponding to this magnetic field can be presented as [2]

$$\mathbf{A} = \mathbf{a}\delta(r), \quad \mathbf{a} = -\pi \int \mathbf{j}(r) r^2 \, d^3 r, \tag{1}$$

where **a** is an anapole moment vector directed along the nuclear spin **I**.

Though the nuclear AM was predicted by theorists long ago [2], it was revealed only recently in an atomic experiment with cesium [3]. The first measurement of AM provided a valuable probe of the relatively poorly understood parity nonconservation in nuclei. Further experimental and theoretical investigations of AM are very important for nuclear physics as well as for the physics of the fundamental interactions.

In this paper we calculate the nuclear spin-dependent P-odd amplitude of the electron–nucleon interaction in atomic ytterbium for the $^1S_0 \to {}^3D_{1,2}$ transitions. Yb was chosen for two reasons. First, parity nonconservation effects are enhanced here because of the closeness of the opposite parity states. This enhacement has been pointed out by DeMille and confirmed by the calculations of the nuclear spin-independent P-odd amplitudes for these transitions [4]. Second, the experiment searching for parity-nonconservation in atomic Yb is in progress at Berkeley [5].

2. Method of calculations and results

The Hamiltonian of the electron-nuclear P-odd interaction can be written as follows:

$$H_{\rm PNC} = H_{\rm si} + H_{\rm sd} = \frac{G_{\rm F}}{\sqrt{2}}\left(-\frac{Q_{\rm W}}{2}\gamma_5 + \frac{\kappa}{I}\vec{\alpha}\mathbf{I}\right)\rho(\mathbf{r}), \tag{2}$$

where $G_{\rm F} = 2.2225 \times 10^{-14}$ a.u. is the Fermi constant of the weak interaction, $Q_{\rm W}$ is the nuclear weak charge, κ is the dimensionless coupling constant, $\vec{\alpha} = \gamma_0\vec{\gamma}$, γ_i are the Dirac matrices, $\mathbf{I}$ is the nuclear spin, and $\rho(\mathbf{r})$ is the nuclear density distribution. Atomic units are used throughout the paper.

We assume that the nucleus is a uniformly charged sphere:

$$\rho(\mathbf{r}) = \frac{3}{4\pi R_0^3}\Theta(R_0 - r),$$

where R_0 is the nuclear radius.

The first term in (2) describes nuclear spin-independent part of electron–nucleus P-odd interaction. In this paper we focus on the second term in (2) and calculate the nuclear spin-dependent amplitude for the ${}^1S_0 \to {}^3D_{1,2}$ transitions. The $M1$ transition ${}^1S_0 \to {}^3D_1$ is strongly forbidden, because $\triangle S \neq 0$ and $\triangle L \neq 0$, where S and L are the total spin and orbital angular momentum of the atom in a given state. The ${}^1S_0 \to {}^3D_2$ transition has a significant electric quadrupole amplitude. Its magnitude was experimentally determined in [5].

The P-odd electron–nuclear interaction “mixes” $E1$ transition to the ($M1$ or $E2$) ${}^1S_0 \to {}^3D_J$ transition. Taking into account only the second term in (2) the corresponding reduced P-odd matrix element can be written in the second order of the perturbation theory as

$$E1_{\rm SD} = \kappa G_{\rm F}(-1)^{2\rm F}\left(\frac{(I+1)(2I+1)(2F+1)}{6I}\right)^{1/2}\begin{Bmatrix} J & 1 & 1 \\ I & I & F \end{Bmatrix}$$
$$\times \sum_n \left((-1)^J \frac{\langle {}^3D_J||\alpha\rho(\mathbf{r})||n\rangle\langle n||E1||{}^1S_0\rangle}{E_{{}^3D_J} - E_n} + \frac{\langle {}^3D_J||E1||n\rangle\langle n||\alpha\rho(\mathbf{r})||{}^1S_0\rangle}{E_{{}^1S_0} - E_n}\right), \tag{3}$$

where E_i is the energy of the state i, F is the final total angular momentum. The initial total angular momentum is $F' = I$.

We need to stress once more that we omitted the spin-independent part of the P-odd Hamiltonian in the expression (3), though for the ${}^1S_0 \to {}^3D_1$ transition this part gives the dominating contribution to the P-odd amplitude. For this reason the measurement of the nuclear spin-dependent P-odd effects in this transition would rely on comparison of the P-odd amplitudes on different hyperfine components of the transition.

As it was noted by Khriplovich, the $^1S_0 \to {}^3D_2$ transition has an advantage for search for nuclear spin-dependent effects, because the nuclear spin-independent part of the Hamiltonian does not contribute here. $H_{\rm si}$ being the scalar operator, mixes only the states with the same total electronic angular momenta. It is easy to see that only intermediate states with $J_n = 1$ give nonzero contribution in expression (3).

The peculiarities of the transitions in question are that in the sum over all intermediate states one needs to account only for three lowest-lying odd-parity states which give nonzero contributions: $^3\mathrm{P}_1^0$(6s6p) (17992 cm^{-1}), $^1\mathrm{P}_1^0$(6s6p) (25068 cm^{-1}), and $^3\mathrm{P}_1^0$(6s7p) (38417 cm^{-1}). (The respective energies [6] are indicated in parenthesis.) According to our estimate all higher-lying states contribute to $E1_{\rm SD}$ less than 1% and we neglect them.

In our calculations we use the combined method of the configuration interaction (CI) and the many-body perturbation theory (MBPT) suggested in [7] for calculation of energy spectra. Subsequently the method was extended to calculations of various observables and was repeatedly used by our group for calculations of several atoms [8]. Here we give only a brief description of the calculation features.

We consider ytterbium as a two-electron atom with the [$1\mathrm{s}^2 \ldots 4\mathrm{f}^{14}$] core. Valence–valence correlations are taken into account by the CI method, while core–valence and core–core correlations are treated within the second order of the MBPT. The latter is used to construct an effective Hamiltonian for the CI problem in the valence space. The calculation is done in the V^N approximation, i.e., the core orbitals are obtained from the Dirac–Hartree–Fock (DHF) equations for a neutral atom (we use the DHF computer code [9]). The basis set for the valence electrons includes 6s, 6p, 5d, 7s, 7p, and 6d DHF orbitals and 8s–15s, 8p–15p, 7d–14d, 5f–10f, and 5g–7g virtual orbitals. The latter were formed in two steps. On the first step we construct orbitals with the help of a recurrent procedure, which is described in [10]. Subsequently we diagonalize the V^N DHF operator to obtain the final set of orbitals. For this orbital basis set the complete CI is made for both even-parity and odd-parity levels.

Results of the nuclear spin-dependent amplitude $E1_{\rm SD}$ calculations are presented in table 1. Summing up, one can say the following.

(1) The nuclear spin of Yb is determined by the external neutron, while the nuclear spin of Cs is determined by the external proton. The calculations showed that the P-odd nuclear spin-dependent amplitude for $^1S_0 \to {}^3D_J$ transitions in Yb is two orders of magnitude larger than in Cs. This is in good agreement with the results of the recently published paper [11]. All this makes Yb a good candidate for the experimental search for the nuclear anapole moment.

(2) In $^1S_0 \to {}^3D_2$ transition nuclear spin-independent part of the P-odd Hamiltonian does not contribute to P-odd transition amplitude, while the P-odd amplitude due

Table 1
Nuclear spin-dependent P-odd amplitudes $E1_{\mathrm{SD}}$ are in units of $(\kappa e a_0 \times 10^{-11})$, A is the number of nucleons in the nucleus.

Isotope	Transition	$F' \to F$	$E1_{\mathrm{SD}}$
$A = 171$	${}^1S_0 \to {}^3D_1$	$1/2 \to 1/2$	−2.75
$I = 1/2$	${}^1S_0 \to {}^3D_1$	$1/2 \to 3/2$	−1.94
	${}^1S_0 \to {}^3D_2$	$1/2 \to 3/2$	9.13
$A = 173$	${}^1S_0 \to {}^3D_1$	$5/2 \to 3/2$	2.82
$I = 5/2$	${}^1S_0 \to {}^3D_1$	$5/2 \to 5/2$	−0.99
	${}^1S_0 \to {}^3D_1$	$5/2 \to 7/2$	−2.85
	${}^1S_0 \to {}^3D_2$	$5/2 \to 3/2$	3.89
	${}^1S_0 \to {}^3D_2$	$5/2 \to 5/2$	−6.79
	${}^1S_0 \to {}^3D_2$	$5/2 \to 7/2$	−8.05

to nuclear spin-dependent electron–nucleon interaction is larger for all hyperfine transitions than for the similar ${}^1S_0 \to {}^3D_1$ transitions.

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