Parity non-conservation with multiply charged ions

Mikhail Kozlov

Petersburg Nuclear Physics Institute
Neutron Research Division

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Plan of the talk

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Comparison with atoms

Energy levels of He-like ions

Close levels of opposite parity

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Outline

PNC Hamiltonian

$$H^{\mathrm{PNC}} = -\frac{G_{\mathrm{F}} Q_{\mathrm{W}}}{2\sqrt{2}} \gamma_5 n(\boldsymbol{r}),$$

where $G_{\mathrm{F}} = 2.2225 \times 10^{-14}$ a.u. is the Fermi constant of the weak interaction, γ_5 is the Dirac matrix, and $n(\boldsymbol{r})$ is the nuclear density normalized as $\int n(\boldsymbol{r}) d\boldsymbol{r} = 1$. The dimensionless constants Q_{W} is known as the weak charge of the nucleus:

$$Q_{\mathrm{W}} = -N + Z(1 - 4\sin^2\theta_{\mathrm{W}}) \approx -N.$$

PNC matrix element

Due to the short-range nature of the interaction H^{PNC} it mixes only one-electron states with $j = 1/2$, i.e. $n_1 s_{1/2}$ and $n_2 p_{1/2}$.

For H-like ion:

$$\langle n_2 p_{1/2} | H^{\mathrm{PNC}} | n_1 s_{1/2} \rangle = \frac{-i\sqrt{2}\, G_{\mathrm{F}} \alpha}{8\pi (n_1 n_2)^{3/2}} Z^4 R(Z) Q_W \sim Z^5 R,$$

where $R(Z)$ is the relativistic enhancement factor,

$R(1) = 1, \quad R(80) \approx 10.$

For neutral atom:

$$\langle n_2 p_{1/2} | H^{\mathrm{PNC}} | n_1 s_{1/2} \rangle = \frac{-i\sqrt{2}\, G_{\mathrm{F}} \alpha}{8\pi (\tilde{n}_1 \tilde{n}_2)^{3/2}} Z^2 R(Z) Q_W \sim Z^3 R.$$

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PNC mixing

PNC effects in atoms and ions appear because of the mixing of the levels of opposite parity. This mixing leads, for example, to an admixture of a negative-parity state ψ_- to a positive-parity state ψ_+ due to the parity nonconserving weak interaction H^{PNC}, $\psi_+ + i\eta\psi_-$, as determined by the first-order perturbation expression

$$i\eta = \frac{\langle -|H^{\mathrm{PNC}}|+\rangle}{E_+ - E_- + \frac{i}{2}\Gamma_-}.$$

When $|E_+ - E_-| \gg \Gamma_-$, coefficient η is real. In neutral atoms the valence energies are roughly independent of Z, and η scales as Z^3R. In MCI the level energies $E_\pm$ are proportional to Z^2 and a typical PNC mixing η again scales as Z^3R.

Comparison of highly charged ions with atoms

- For ions PNC amplitudes grow faster with Z.
- Energy splittings between levels of opposite parity also grow with Z.
- Typical PNC mixings grow as Z^3 for both atoms and ions.
- For hydrogen-like ions the levels of opposite parity $ns_{1/2}$ and $np_{1/2}$ are anomalously close because of the "accidental" degeneracy. The splitting, caused by the Lamb shift, grows rapidly with Z ($\sim Z^4$).
- For He-like ions the levels of opposite parity can cross at some Z. That can cause huge additional enhancement of the PNC mixing (Gorshkov & Labzowski).

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Configurations $1s2s$ and $1s2p$

- The levels ${}^1S_0(1s2s)$ and ${}^3P_1(1s2p)$ cross at $Z \approx 32$. This is a $\Delta J = 1$ crossing and PNC mixing is caused only by the nuclear-spin-dependent PNC interaction (Gorshkov & Labzowski, 1974).
- The levels ${}^1S_0(1s2s)$ and ${}^3P_0(1s2p)$ cross at $Z \approx 65$ and $Z \approx 90$ (Andreev et al, 2003).
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Configurations $2s^2$, $2p^2$, and $2s2p$

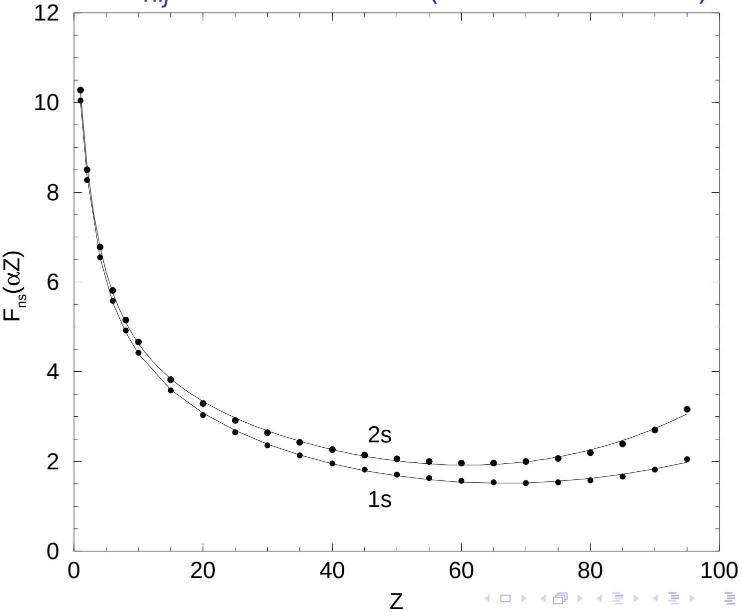
- The energies of the $2l2l'$ states are determined by diagonalization of the effective Hamiltonian in the $n = 2$ subspace.
- The single-electron part of this Hamiltonian includes hydrogenic Dirac orbital energies and the Lamb shift.
- The two-electron part of the Hamiltonian matrix is presented as a double expansion in parameters $1/Z$ and αZ (Braun, Gurchumelia, & Safronova).
- We use first three terms of this expansion of order Z, $Z(\alpha Z)^2$, and Z^0.

Lamb shift for H-like ions

The Lamb shift is known to be essential for the level crossings within the $1s2l'$ manifold (Gorshkov & Labzowski). The same is also true for the $2l2l'$ states.
By factoring out the main dependence on Z and the principal quantum number n, the Lamb shift for the hydrogenic orbital nlj is written as

$$\delta E_{nlj} = \frac{Z(\alpha Z)^3}{\pi n^3} F_{nlj}(\alpha Z).$$

Function F_{nlj} for H-like ions (Johnson & Soff)



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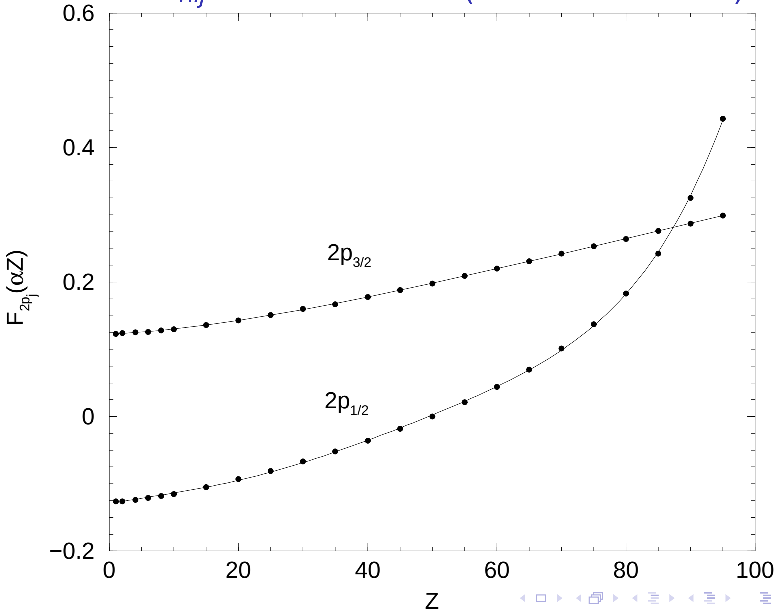

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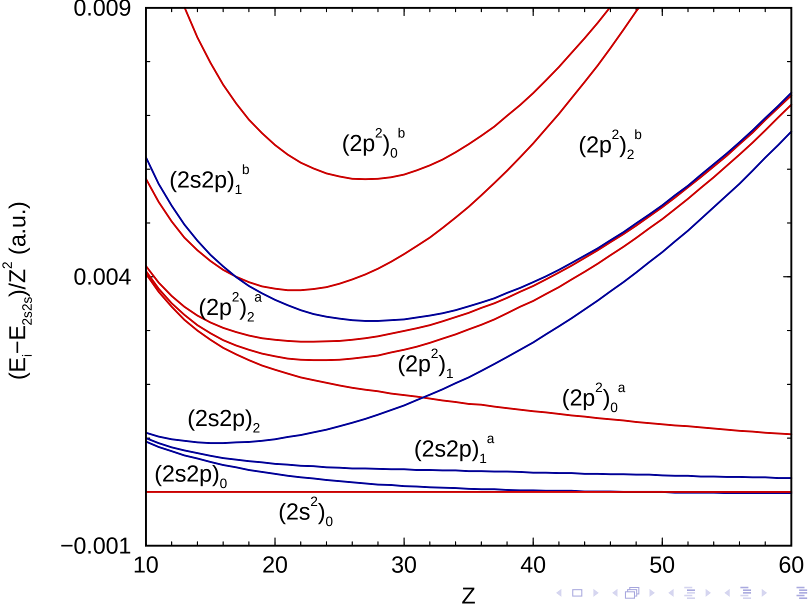

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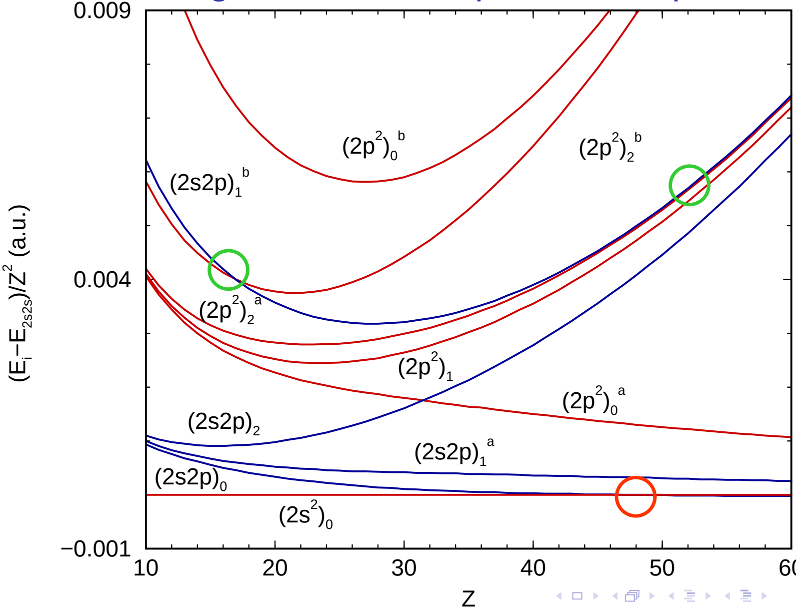

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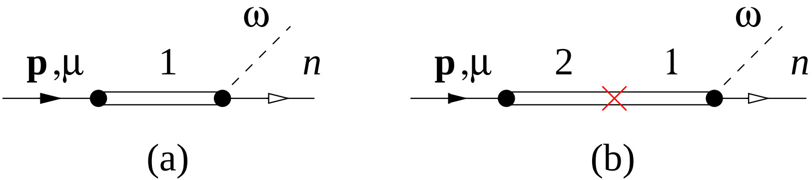
Dielectronic recombination (DR): e + H(1s) → He*



DR cross section consists of parity conserving (PC) and PNC terms, $\sigma = \sigma^{\rm PC} + \sigma^{\rm PNC}$. PNC asymmetry $\mathcal{A}$ is defined as:

$$\mathcal{A} = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-} \simeq \left.\frac{\sigma^{\rm PNC}}{\sigma^{\rm PC}}\right|_{\mu=1},$$

where $\sigma^\pm$ are the cross sections for positive and negative helicity, $\mu \equiv \boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}} = \pm 1$.

PC cross section

Contribution of the resonance k to the PC part of the DR cross section has the standard Breit-Wigner form:

$$\sigma_k^{\mathrm{PC}}(\varepsilon) = \frac{\pi}{4p^2} \frac{\Gamma_k^{(r)}\Gamma_k^{(a)}}{(E_{1s} + \varepsilon - E_k)^2 + \frac{1}{4}\Gamma_k^2},$$

where E_k and Γ_k are the energy and the total width of the resonance. The latter is the sum of the autoionizing and radiative widths: $\Gamma = \Gamma^{(a)} + \Gamma^{(r)}$.

Autoionizing and radiative widths

Radiative width of the $(2s2p)_0 \equiv (-,0)$ and $(2s^2)_0 \equiv (+,0)$ states in the non-relativistic approximation is given by:

$$\Gamma^{(r)}_{-,0} = \left(\frac{2}{3}\right)^8 \alpha^3 Z^4 = 1.517 \times 10^{-8} Z^4,$$

$$\Gamma^{(r)}_{+,0} = 2\left(\frac{2}{3}\right)^8 \alpha^3 Z^4 \left(1 - C^2_{ss}\right),$$

where the coefficient C_{ss} defines contribution of the configuration $2s^2$ to the state $|+,0\rangle$.
In the same approximation $\Gamma^{(r)}$ is independent on Z,

$$\Gamma^{(a)}_{-,0} = 0.0104, \quad \Gamma^{(a)}_{2s^2} = 0.00496.$$

Note that for $Z \gg 30$, $\Gamma_k \approx \Gamma^{(r)}_k \gg \Gamma^{(a)}_k$.

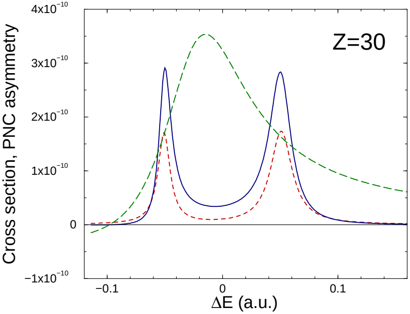
PNC cross section

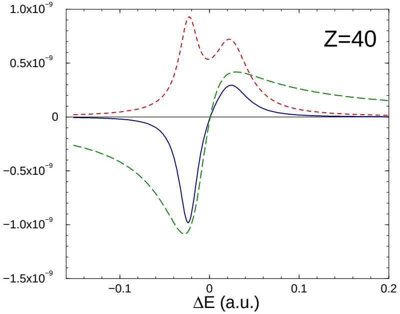
Contribution of the resonance k to the PNC part of the DR cross section takes the form:

$$\sigma_k^{\mathrm{PNC}}(\varepsilon) = (\boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}})\, \sigma_k^{\mathrm{PC}} \sqrt{\frac{\Gamma_i^{(a)}}{\Gamma_k^{(a)}}} \times \Re\left\{ e^{i(\delta_k - \delta_i)} \frac{2i\langle k|H^{\mathrm{PNC}}|i\rangle}{E_{1s} + \varepsilon - E_i + \frac{i}{2}\Gamma_i} \right\},$$

where δ_i and δ_k are the Coulomb scattering phases.

PC & PNC DR cross sections for $Z = 30, 40$

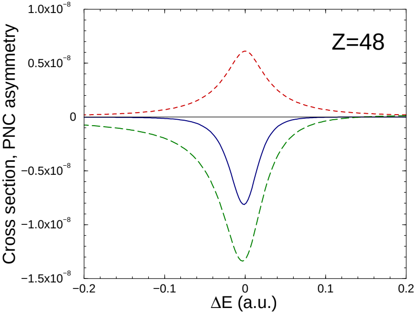


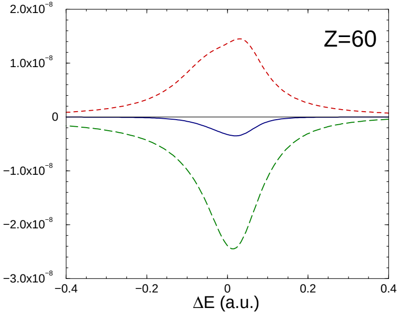


Solid blue: $10^3 \times \sigma^{\rm PNC}$;
Dashed red: $10^{-7} \times \sigma^{\rm PC}$;
Dashed green: PNC asymmetry $\mathcal{A}$.

Solid blue: $10^3 \times \sigma^{\rm PNC}$;
Dashed red: $10^{-6} \times \sigma^{\rm PC}$;
Dashed green: PNC asymmetry $\mathcal{A}$.

PC & PNC DR cross sections for $Z = 48, 60$





Solid blue: $10^3 \times \sigma^{\mathrm{PNC}}$;
Dashed red: $10^{-5} \times \sigma^{\mathrm{PC}}$;
Dashed green: PNC asymmetry $\mathcal{A}$.

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PNC measurement feasibility functions

Let us estimate the sensitivity requirements for an experimental apparatus capable of observing the PNC asymmetry. The number of counts in an experiment with a fully polarized electron beam with positive helicity is given by:

$$N_{\pm} = j_e N_i t \epsilon \sigma^{\pm} \equiv I \sigma^{\pm},$$

where j_e is the electron flux, N_i is the number of target ions, t is the acquisition time, and ϵ is the detection efficiency.

Monoenergetic electron beam

For a beam or target with polarization P, to detect the PNC asymmetry, the difference between the counts, $P|N_+ - N_-|$ should be greater than statistical error, $\sqrt{N_+ + N_-}$, which gives:

$$I(\varepsilon) > \frac{\sigma^+(\varepsilon) + \sigma^-(\varepsilon)}{P^2[\sigma^+(\varepsilon) - \sigma^-(\varepsilon)]^2},$$

For $P = 1$ the feasibility of the experiment on ions with nuclear charge Z depends on the functions:

$$F(Z) = \min_\varepsilon \left\{ \frac{\sigma^+(\varepsilon) + \sigma^-(\varepsilon)}{[\sigma^+(\varepsilon) - \sigma^-(\varepsilon)]^2} \right\}.$$

The beam with wide energy distribution

If the electron energy spread in the beam is greater than the resonance spacing and widths, then the flux j_e should be replaced by the flux density $dj_e/d\varepsilon$. The counts $N_\pm$ are obtained by integrating over the electron energy and the effect can be detected if

$$I_{\rm av} > \int (\sigma^+ + \sigma^-) d\varepsilon \left[\int (\sigma^+ - \sigma^-) d\varepsilon \right]^{-2} .$$

Obviously, now the feasibility depends on the function:

$$F_{\rm av}(Z) = \int (\sigma^+ + \sigma^-) d\varepsilon \left[\int (\sigma^+ - \sigma^-) d\varepsilon \right]^{-2} .$$

The resonance strength

If we introduce the resonance strength:

$$S_k = \int \sigma_k^{\mathrm{PC}} d\varepsilon = \frac{\pi^2}{2p^2} \frac{\Gamma_k^{(r)} \Gamma_k^{(a)}}{\Gamma_k},$$

and the PNC strength:

$$S_{1,2}^{\mathrm{PNC}} \equiv \sum_{k=1}^{2} \int \sigma_k^{\mathrm{PNC}} \Big|_{\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}=1} d\varepsilon,$$

we can present the feasibility function in a form:

$$F_{\mathrm{av}}(Z) = \tfrac{1}{2}(S_1 + S_2) / \left(S_{1,2}^{\mathrm{PNC}}\right)^2 .$$

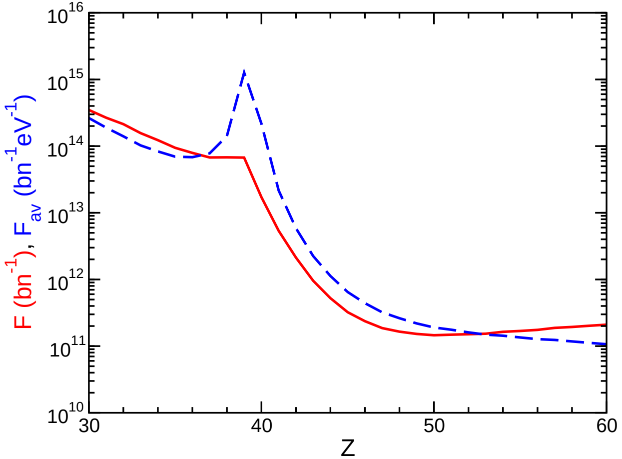
Meaning of the functions F and F_{av}

$$F(Z) = \min_{\varepsilon} \left\{ \frac{\sigma^{+}(\varepsilon) + \sigma^{-}(\varepsilon)}{[\sigma^{+}(\varepsilon) - \sigma^{-}(\varepsilon)]^2} \right\} .$$

$$F_{\text{av}}(Z) = \tfrac{1}{2}(S_1 + S_2) / \left(S_{1,2}^{\text{PNC}}\right)^2 .$$

- Note that F^{-1} and F_{av}^{-1} have the same dimensions as a cross section and a resonance strength respectively.
- For an experiment to be able to observe predicted PNC effect in KLL dielectronic recombination resonances, it would have to be able to detect a cross section as small as F^{-1} or a resonance strength as small as F_{av}^{-1}.

PNC measurement feasibility functions



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Proposed PNC experiments in MCI

- **PNC effect in the Auger emission from the He-like uranium** *(Pinzola, 1993).* PNC asymmetry $\sim 10^{-7}$ is caused by the mixing of the states $(2s^2)_0$ and $(2s2p)_0$ with $J = 0$. That estimate neglected the radiative widths of the levels, which for $Z \gtrsim 50$ exceed the level spacing.
- **PNC asymmetries in radiative transitions in He-like uranium.** *Schafer et al.* considered two-photon $E1$-$M1$ transition $2\,^3P_0^o \to 2\,^1S_0$, separated by 1 eV. Here PNC mixing is $|\eta| \sim 5 \times 10^{-6}$, but one needs laser intensity above 10^{21} W/cm^2 to observe this transition.
- **PNC asymmetry of the photon angular distribution in He-like gadolinium.** The asymmetry for the hyperfine-quenched transitions $2\,^1S_0 \to 1\,^1S_0$ is 4×10^{-4}. Although this value is large, the number of events necessary to measure the effect is $\sim 10^{18}$ *(Nefedov et al. (2002)).*

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Comparison of different proposals

Typical PNC experiments with MCI require:

- Spin-polarization of the ions.
- Observation of the (highly) forbidden transitions.
- Observation of the circular polarization of γ-quanta.

In our proposal:

- Spin-polarization of either ions, or electrons is required.
- Observation of a normal, rather then weak DR resonance.
- The PNC asymmetry does not include γ-quanta.

All proposals to observe PNC effects in MCI are experimentally challenging.

Observation of PNC effects in MCI will give a theoretically clean test of the Standard Model at low energies and in the strong electric fields.

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